Studies of Coalescence and Phase Separation in Thick Dispersion Bands

A refractive index matching technique has been developed to study the behavior and coalescence phenomena of drops in closely packed dispersion bands. The frequency of drop-drop coalescence has been measured and correlated, and a mathematical model, based on film rupture thickness, has been written for solution by computer to predict dispersion band thickness as a function of inlet drop size, flow rate, and the physical properties of the system.

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SCOPE

The separation of two immiscible liquid phases is an important chemical engineering operation in liquid extraction and effluent treatment processes. In this operation, drops of the dispersed phase accumulate and pack closely together to form a dispersion band in which the continuous phase is trapped in the interstices between the drops. The rates of drainage of the continuous phase and the ensuing coalescence process determine the thickness of this band and, frequently, the capacity of the process. Therefore, for the design of such equipment, it is necessary to be able to predict the thickness of the dispersion band in phase separation operation, and in recent years many investigations have been initiated to establish the characteristics of dispersion bands. In all these studies, drop behavior inside the band has been assumed to be identical with that seen at the periphery because the opacity of the closely packed dispersion prevented observations of the drops inside. Hence in order to study the coalescence behavior inside such a dispersion band, systems have been chosen in which the refractive index of the dispersed and continuous phases are the same, and then one or more colored drops have been fed into the transparent band and their behavior followed. Care was taken to ensure that the colored drops had identical properties with all the other drops in the band; consequently Red Oil O, which Kintner (1968) reported to be nonsurface active, was used to produce discrete colored drops, and these were studied as they passed through the band. A second method used in this investigation was to generate the color in the drops, in situ, at particular positions in the band by dissolving a small quantity of a phototropic dye in one or other of the phases and irradiating the particular section of the dispersion band to generate the colored drops. Claude and Rumpf (1953) reported that most spirans are sensitive phototropic dyes and 1.3.3 trimethylindolino-6′ nitrobenzopyrylspiran was chosen for this work because a 1.0% solution of this compound was sufficient to produce an intense color change when irradiated with ultraviolet light. The color was sustained for about 10 min. after exposure to the u.v. light. After this period it reverted to its colorless form but could be irradiated repeatedly so that solutions containing the dye could be circulated through the apparatus continuously.

The Christiansen Effect (1884) was also employed to identify drops in the dispersion band. Christiansen observed that the refractive indices of the phases in a dispersion are only matched at a particular wave length. When such a dispersion is exposed to light of different wave length the interface between the phases can be seen. Therefore, the refractive index of each phase had been matched against the sodium D line so that the dispersion was transparent to yellow light. Then a beam of red light from a gas laser was passed through the coalescence band, and since the refractive index of the phases will not be matched for all wavelengths, the drops in the beam become opaque and are easily seen and their behavior followed.

The techniques described above were used to observe the phase separation processes occurring in thick dispersion bands in order to explain the mechanism of the drainage of the continuous phase film and the coalescence of the drops in these bands. These mechanisms were then applied to develop a model to predict the thickness of a dispersion band of the kind encountered in liquid extraction and effluent treatment processes as a function of flow rate, drop size, and the physical properties of the system.

CONCLUSIONS AND SIGNIFICANCE

A new technique has been developed to study the phase separation processes occurring inside thick dispersion bands of the kind found in the coalescence section of an extraction column and in effluent treatment operations. It is based on matching the refractive index of each phase so that the dispersion becomes transparent; then a colored drop is inserted or generated so that it has been possible to study drop behavior inside dispersion band.

The experimental observations revealed that there were three distinct zones in this type of dispersion band: A flocculating zone, a packing zone, and an interfacial coalescence zone; the significant phase separation processes determining the thickness of the dispersion band occur in the packing zone. In this zone the drops pack closely together to take the shape of regular pentagonal dodecahedra. The thickness of the dispersion band depends on the rate of drainage of the continuous phase film from in

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between adjacent faces of these dodecahedral shaped drops and on the thickness of this film when it ruptures, thus permitting interdrop coalescence. The thickness of the continuous phase film at rupture was estimated experimentally and found to be dependent on the physical properties of the system. For the systems studied it varied between 1.5 and 2.5 microns.

A mathematical model has been developed to describe the coalescence processes occurring in dispersion bands and to predict the thickness of the band. It has been based on the size and shape of the drops in the band, the frequency of interdrop coalescence, which was found to be a function of the physical properties of the system, drop size, and the thickness of the continuous phase film at rupture. The model was tested by writing an appropriate computer program and comparing the predicted dispersion band thickness with the experimental results, and, in all cases, the agreement was very good. Furthermore, it has been possible to photograph the shape of the plateau border through which the continuous phase drains and to estimate the radius of curvature of the plateau border. These radii have been compared with calculated values obtained from the model and the agreement is also found to be very good, thereby further confirming the validity of the model and the description of the mechanism of the coalescence process in these bands.

SYSTEMS STUDIED

The systems studied in this investigation were aqueous solutions of glycerine with refractive indices in the range 1.333 to 1.47 for the aqueous phase and either *iso*-octane *n*-hexane, amyl acetate, or ethyl acetate, whichever had the same refractive index as that of a glycerine solution as the other phase. Details of the systems used are given in Table 1.

The spiran used as the phototropic dye was insoluble in the glycerine phase. It was therefore dissolved in the organic phase to make a 1.0% solution, and its surface activity was assessed by comparing the interfacial tension of each system in the excited and unexcited state. No difference was detected. In addition, the time taken for a dispersion band of a given thickness containing drops of a known size to collapse after the dispersed phase flow had been stopped was also measured when the band had been radiated with u.v. light and this time was compared with the time of collapse of the same system that had not been excited. The following results were obtained for the system isobutylacetate in aqueous glycerine. For a band 11.5-cm thick the time of collapse was 15.9 s in the excited and 15.7 s in the unexcited state; also a band of the same system 17.8 cm thick the times of collapse were respectively 19.8 and 20 s.

EXPERIMENT

The apparatus for this study is illustrated schematically in Figure 1. It consisted of a glass column (1) 5.0 cm in diameter, 60.0 cm long to which was attached a 5.0 cm to 7.5 cm reducer to accommodate the dispersed phase distributor. The column was connected to the dispersed phase reservoir (2) via a pump (3), a sintered glass filter (4), constant head tank (5), and the rotameters (6) as shown. The pump was an all glass centrifugal pump possessing P.T.F.E. seals and the sintered glass filter was inserted to trap any scum and solids. The dispersed phase reservoir was constructed from a glass pipe section 22.5 cm in diameter, 22.5 cm long with stainless steel flanges attached to each end. The constant head tank was similarly constructed from a glass pipe section 15.0 cm in diameter, 15.0 cm long, and the interconnecting pipework was entirely of glass of 1.5 cm nominal bore. The distributor was constructed of stainless steel and contained a distribution plate with 56 carefully prepared nozzles so that drops of the dispersed phase of uniform size could be produced. In addition, a separate inlet tube was connected to the central nozzle so that one or more colored drops of the same size could be injected into the column. The distribution plates were interchangeable so that dispersions with drops of different sizes could be studied.

A typical experiment was performed by thoroughly cleaning the apparatus, first by flushing with tap water, soaking in a concentrated solution of decone for 24 hours, draining, and flushing again with distilled water. Following this, the column was partially filled with the continuous phase to a predetermined height and the reservoirs were filled with the dispersed phase. The pump was started so that the dispersed phase formed a dispersion band and after coalescence filled the remainder of the column. The coalesced dispersed phase was circulated continuously through the apparatus and its flow rate was adjusted to produce a dispersion band of suitable thickness. The thickness of the band was determined by injecting a colored drop from the special nozzle in the distributor, or, when one of the phases contained the phototropic dye, by exposing the band to u.v. light from a mercury arc lamp so that it became blue in color. After a number of experiments had been conducted, the location of the band and its thickness could be estimated from the dispersed phase flow rate although, because of the 'matched' refractive index of the phases, the dispersion band could not be seen in the column.

When steady state had been attained, one or more colored drops were injected into the band and their behavior photographed as they passed through, using a high speed camera. Alternatively the spiran was dissolved in the organic liquid which could be either the dispersed or continuous phase. The dispersed phase was again circulated until steady state conditions had been attained, then the band, or parts of it, were exposed to u.v. light for a few seconds. This produced a layer,

TABLE 1. DETAILS OF SYSTEMS USED IN INVESTIGATION

System	Dispersed phase	Continuous phase	Refractive index	Viscosity cp		Density g/cm³		Interfacial tension
				Disp. μ_d	Cont. μ_c	Disp. ρ_d	Cont. ρ_d	γ dynes/cm
I	* Amylacetate	52% wt. G	1.460	1.350	8.00	0.870	1.140	14.2
II	* Ethylacetate	37% wt. G	1.372	0.455	3.60	0.900	1.090	7.0
III	* Iso octane	48% wt. G	1.395	0.433	5.45	0.692	1.122	35.6
IV	* Hexane	37% wt. G	1.372	0.326	3.60	0.660	1.090	34.5
V	* 17% wt. G	Diethylether	1.355	1.60	0.233	1.042	0.713	10.0
VI	* 46% wt. G	Isobutylacetate	1.392	5.25	0.732	1.118	0.870	12.5
VII	Isopropylether	27% G	1.367	0.470	2.15	0.730	1.065	15.0

* Systems that have been investigated.

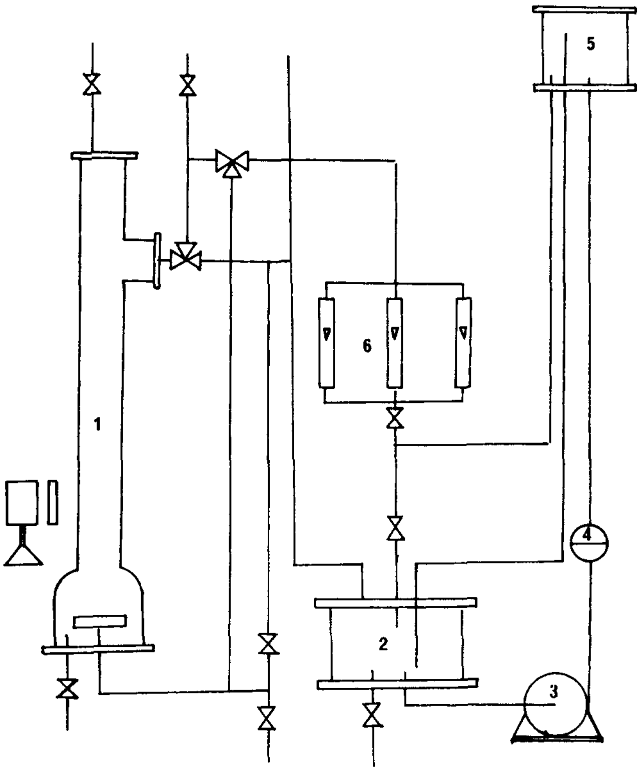


Fig. 1. Flow diagram of apparatus.

or cluster, of blue drops and their progress through the bed similarly photographed.

From these experiments the mean drop residence time and the frequency of coalescence between a drop and the wall, a pair of drops and the drop and the interface was obtained, and these have been reported in detail as a function of band thickness by Allak (1973).

RESULTS

Experiments were carried out with a number of distributors having nozzles of uniform size in the range 1.0 mm to 4.0 mm, and the dispersed phase flow rate was controlled to produce uniform sized drops. The drop size was estimated from photographs of the colored drops entering the dispersion band, and these were compared with the drop size predicted from the procedures recommended by Meister and Schiele (1968). In all cases the agreement between the estimated and predicted value was very good.

Observation of the bed when steady state had been attained revealed that there were three zones in the dispersion band. These are illustrated in Figure 2, thus there is:

1. A flocculating zone at the droplet entrance. When the drops enter the bed they are jostled about with the result that, although they remain spherical, they arrange themselves in the most compact way and trap the continuous phase in the interstices. This is the flocculating zone and the dispersed holdup is probably close to 0.74, that is, the so-called "Oswaldt's ratio."
2. A packing zone. As the drops pass into the band they are squeezed together extruding the continuous phase. Figure 3 shows three colored drops which are surrounded by transparent drops inside the band. There it will be seen that these drops are deformed into the shape of a pentagonal dodecahedra, the faces of which are reasonably flat. The continuous phase is squeezed from in between these faces into plateau borders which form a drainage network throughout this zone. Considerable interdrop coalescence occurs in this zone.
3. Interface coalescing zone. This zone is one or two drops thick in which the drops are still dodecahedra in shape but tend to coalesce with the interface more than with each other.

The thickness of the dispersion band depended on the inlet drop size and the dispersed phase flow rate. A typical plot showing the relationship between the variables and the band thickness is presented in Figure 4 for the system *n*-hexane-aqueous glycerine. The curves in this figure are similar in shape to results reported by other workers and confirm that the band thickness increases as the inlet drop size decreases and the dispersed flow rate increases. The experimental results for all the systems studied were correlated by a dimensional analysis and the correlation in terms of dimensionless groups is presented in Figure 5. It

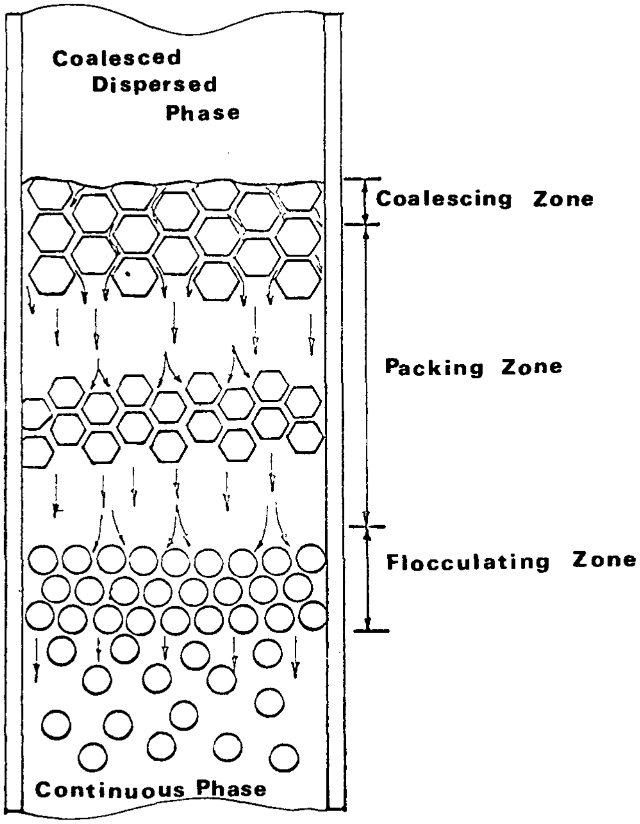


Fig. 2. Illustration of zones in a closely packed dispersion band.

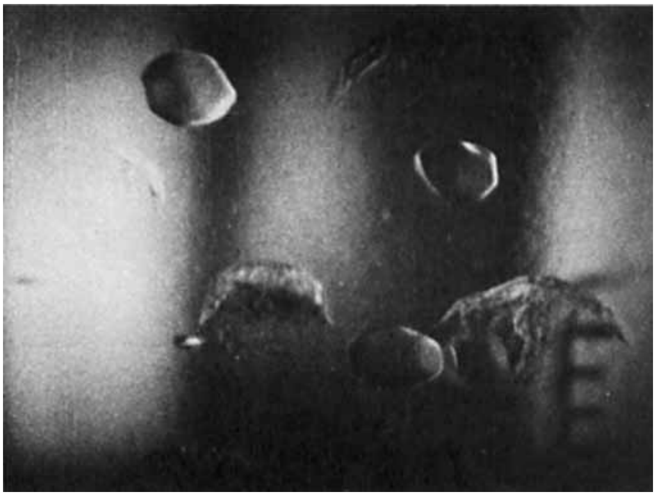


Fig. 3. Photograph of three colored drops in a transparent dispersion band.

was found that 85.0% of the results obtained are within ± 5.0% of the correlating line and 97.5% are within ± 10.0%. It will be seen that the predominant group in this correlation is the capillary number $\left(\frac{\mu_c V_d}{\gamma}\right)$ suggesting that the drainage of the continuous phase film controls the thickness of the dispersion band.

The packing zone extends over most of the dispersion band, and in this zone drop coalescence occurs with the walls of the vessel in addition to interdrop coalescence between pairs of drops. The frequency of drop-wall coalescence was found to be remarkably constant throughout the dispersion band, but was different for each system. The results for the system *n*-hexane-aqueous glycerine are given in Figure 6 where it can be seen that for a dispersion band thickness greater than 6.0 cm the frequency of drop-wall coalescence is constant at 15.0%. Similar results were obtained with the other systems. The frequency of interdrop coalescence varied with band thickness and position in the band, and the results for the *n*-hexane system are shown in Figure 7. The frequencies shown in this figure were estimated from repeated observations of interdrop coalescence at different planes in a band of thickness 29.0 cm and are the mean of 75 repeated experiments on a dispersion band of this thickness. The shape of this frequency curve is typical of the experimental results obtained from the other systems (see Allak, 1973) and gives an insight into the coalescence processes occurring in the band. Thus it will be seen that the frequency of interdrop coalescence is periodic and increases as the drops pass through the band. That is, at a particular plane in the band the drainage of the continuous phase will have proceeded to such an extent that the film between adjacent drops ruptures and interdrop coalescence ensues. There will be slight differences in the film thickness in between different drops in a layer so that coalescence occurs only between the drops separated by the thinnest film. This interdrop coalescence disturbs the drops around the coalescing pair with the result that the film surrounding the neighboring drops is partly regenerated by flow from above and tension in the film from the lower layers. Consequently, the film between the drops is renewed, stabilized temporarily, and must drain again to a critical thickness before further coalescence can occur. During this period this layer continues to pass through the band and is recorded as a region in which the frequency of interdrop coalescence is low. When the film has again drained to a critical thickness further drop-drop coalescences occur, and these processes are repeated until all of the continuous phase trapped at the entrance of the band has been drained. At this point interdrop coalescence is complete and an interface is formed. This point is manifested by the drop-interface coalescence zone.

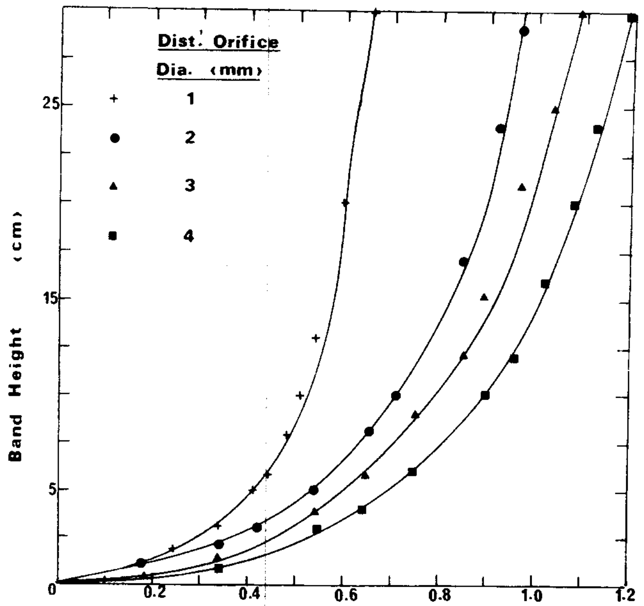


Fig. 4. Band height increase as a function of dispersed phase flow rate *n*-hexanse in aqueous glycerine.

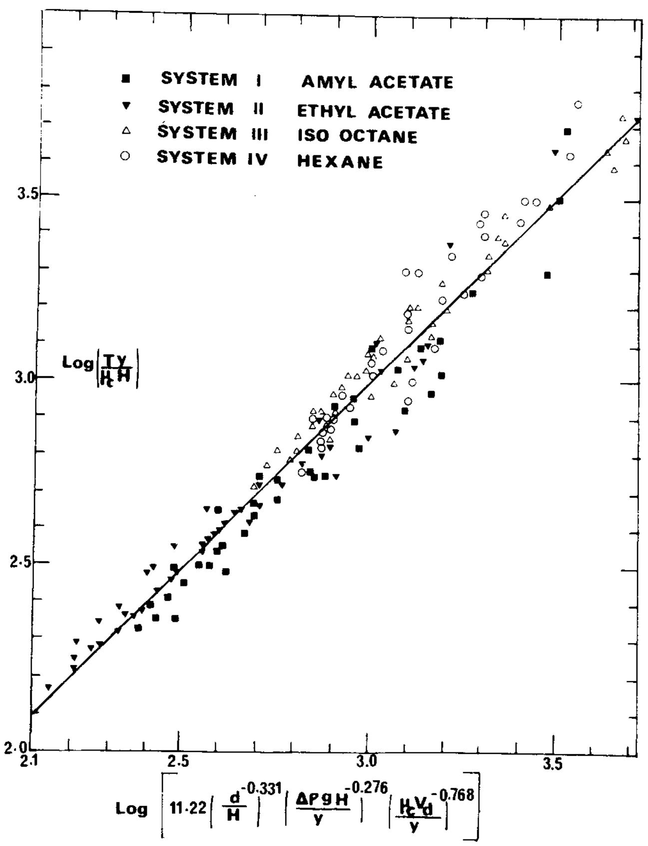


Fig. 5. Correlation of variables of dimensional analysis.

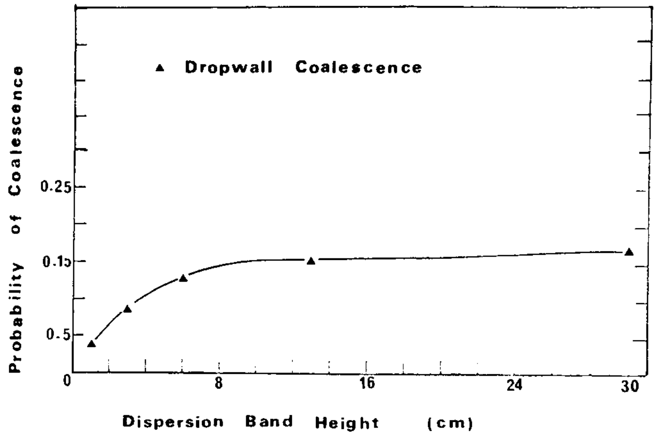


Fig. 6. Probability of dropwall coalescence for the system *n*-hexane.

The frequency of interdrop coalescence in the packing zone has been stated to depend on the drop size, the band thickness, and the rate of drainage of the continuous phase in the film between adjacent drops and through the plateau borders. This frequency can be expressed through the capillary number as shown above in the dimensional analysis.

The resulting correlation was

$$\lambda = 1 - 0.559 \left(\frac{\gamma}{\mu V_d}\right)^{0.054} \left(\frac{d}{H}\right)^{0.19} \qquad (1)$$

where λ represents the frequency, or probability, of coalescence expressed as a fraction of the total number of drops of mean diameter d that coalesce in a plane in the band at a depth H from the entrance to the flocculating zone. A comparison of the experimental and predicted coalescence frequencies for all the systems studied is presented in Figure 8. There it will be seen that there is some scatter between the predicted and experimental results, but because of the periodic frequency of the coalescences and the smoothing necessary in developing the correlation, the agreement is considered to be quite good and is satisfactory for analysis of the phase separation process.

It has been postulated above that interdrop coalescence occurs in the packing zone of the dispersion band when the continuous phase film in between the pentagonal faces of adjacent drops, drains to a critical thickness. This was substantiated by examination of high speed cine films of the dispersion band that contained the phototropic dye in the continuous phase which had been irradiated with u.v. light. It was noticed that as the film thinned the intensity of the blue color decreased proportionately and disappeared immediately before coalescence. This suggests that when the film cannot sustain the blue color it has been reduced to its critical thickness and rupture ensues.

It is not possible to measure the film thickness in the dispersion band, but the thickness of the film that is just unable to sustain the blue color of the phototropic dye was estimated as follows.

Three cells of thickness 50.0, 25.0, and 7.0 microns were used to measure the absorption of light be the continuous phase in the excited and unexcited state in a u.v. absorptiometer, and the absorption was plotted versus cell thickness for the system isobutyl acetate-aqueous glycerine. Extrapolation of the two curves resulted in their intersection at 2.0 microns, suggesting that this is the critical film thickness. This thickness was confirmed by placing the continuous phase with the phototropic dye in between two optical flats, irradiating with u.v. light in the absorptiometer, then reducing the thickness until the color just disappeared, and then measuring the thickness between the optical flats by counting the interference fringes. The film thickness was confirmed to be 2 microns for the system isopropyl acetate, and this was concluded to be a reasonable estimate of the critical film thickness. This, together with the above correlation of probability or frequency of interdrop coalescence, form the basis for a model to predict the thickness of dispersion bands.

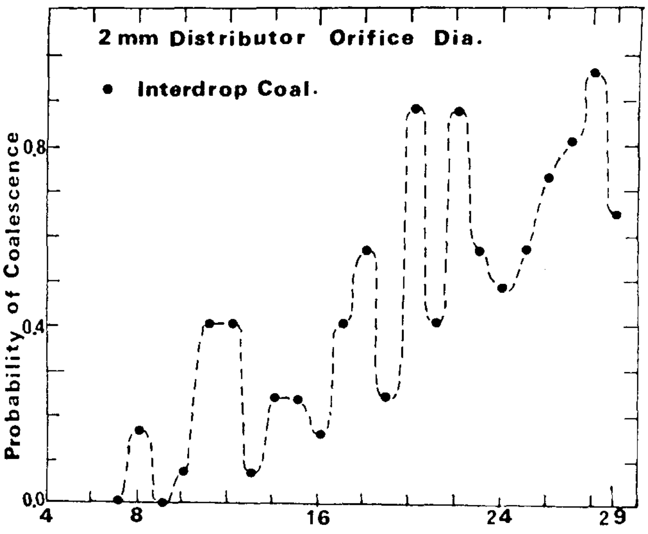


Fig. 7. Probability of interdrop coalescence for the system *n*-hexane.

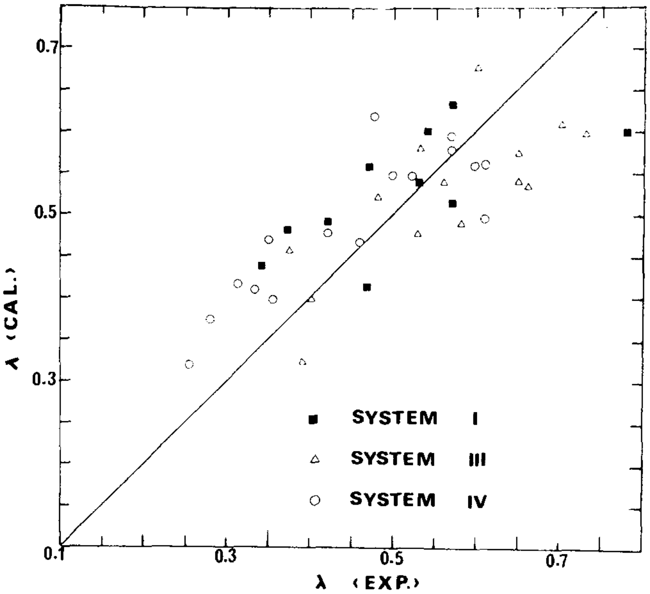


Fig. 8. Comparison between experimental and calculated probability of interdrop coalescence.

MODEL

The description of the drainage process and interdrop coalescence in the packing zone of a dispersion band has been presented above. Therefore consider the drainage of the continuous phase film from inbetween two adjacent faces of a pair of drops. It has been shown that the faces are flat and parallel to each other. Then a force balance gives

$$\frac{\partial P}{\partial r} - \mu \frac{\partial^2 U}{\partial Z^2} = 0 \qquad (2)$$

Assuming laminar flow in the film this gives the rate of thinning in terms of the forces acting to be

$$-\frac{dh}{dt} = \frac{2Fh^3}{3\pi\mu R^4} \qquad (3)$$

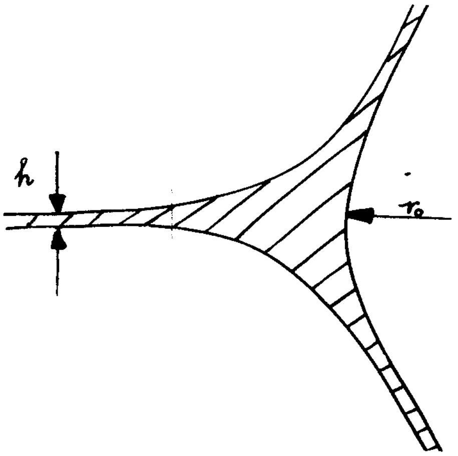
The liquid in the film drains into the plateau borders. On the basis that the drop becomes pentagonal dodecahedra in shape in the dispersion, each drop will have 12 faces. There are 5 edges per face and 10 plateau borders per drop. The length of a plateau border $l = 0.41d$ (for regular dodecahedra). Area of face (pentagon) $1.7205l^2 = 0.29d^2$. The number of plateau borders per unit volume of dispersion is

$$\frac{10\phi_d}{\frac{\pi}{6} d^3} = \frac{60.0\phi_d}{\pi d^3} \qquad (4)$$

The probability of a plateau border being inclined at an angle θ and $(\theta + d\theta)$ to the horizontal is $\cos\theta \cdot d\theta$. Then, the number of plateau borders inclined at an angle between θ and $(\theta + d\theta)$ per unit volume is N, where

$$N = \frac{60\phi_d}{\pi d^3} \cos\theta \cdot d\theta \qquad (5)$$

The vertical projection of a plateau border of length l inclined at an angle θ to the horizontal is $l\sin\theta = 0.41d\sin\theta$, and the probability of it intersecting a horizontal plane per unit height of dispersion is $=0.41d\sin\theta$. The sectional area of a plateau border is



$$Ap = 0.1612r_0^2 + 1.732r_0h$$
(Lenard and Lemlick 1965a)
$$\simeq 0.1612\, r_0^2$$

since $h \simeq 2.0 \times 10^{-4}$ cm and the rate of flow through the plateau border is

$$U = \frac{D^2r_0^2}{2K\mu} \cdot \frac{\Delta P}{\Delta l} \quad \text{Lenard and Lemlick (1956b)} \qquad (6)$$

where $D = 0.205$ and $K = 6.43$. Then the volumetric flow rate $q = UA_p$, that is,

$$q = \frac{0.0065r_0^4}{12.86} \cdot \frac{\Delta P}{\Delta l}$$

For a plateau border inclined at an angle θ

$$\frac{\Delta P}{\Delta l} = \phi_d \rho g \sin\theta$$

and the total flow rate of continuous phase draining through the plateau borders is Q_d where

$$Q_d = \int_0^{\pi/2} (0.000525r_0^4 \rho g \phi_d \sin\theta)\,(0.41d\sin\theta)\left(\frac{60\phi_d}{\pi d^3}\cos\theta\right) d\theta$$

$$Q_d = \frac{0.0041r_0^4 \rho g \phi_d^2}{\mu d^2} \qquad (7)$$

At steady state

Downflow through plateau borders	=	Continuous phase carried up in dispersion band

Upward Flow of Continuous Phase

Rate of drops entering band per unit area of band $= N' = 6V/\pi d^3$ where V is the superficial velocity. Volume of liquid attached to each drop $= 12 \times 0.29d^2 \cdot \frac{h}{2}$. Then upward flow of continuous phase Q_u is

$$Q_u = \frac{6V}{\pi d^3}\left(12 \times 0.29d^2 \times \frac{h}{2}\right) = \frac{3.32Vh}{d} \qquad (8)$$

Also

$$Q_u = \left(\frac{1-\phi}{\phi}\right)V \qquad (9)$$

From Equations (8) and (9),

$$h = \left(\frac{1-\phi}{3.32\phi}\right)d \qquad (10)$$

Drainage Time

In order to integrate Equation (3) the force F is evaluated as follows: Since $Q_d = Q_u$ from Equations (7) and (8)

$$r_0 = \frac{3 \times 3.32Vh\mu d}{0.0041\rho g \phi^2} \qquad (11)$$

and from Laplace's equation

$$\Delta P = \gamma \frac{1}{r_0} + \frac{1}{r_1} = \frac{\gamma}{r_0}$$

and $F = \Delta Pa$ where $a = 0.29d^2$, so that

$$F = \frac{0.29d^2\gamma}{r_0} = 0.29d^2\gamma\left(\frac{0.0041\rho g\phi^2}{9.96Vh\mu d}\right)^{1/4}$$

and Equation (3) becomes

$$\frac{dh}{dt} = -\,1.04 \times 10^{-2}h^{0.25}\left(\frac{\rho g\gamma\phi}{\mu^5d^9}\right)^{1/4} \qquad (12)$$

Integrating between $t = 0$ and $h = h_1$
and
$$t = t \quad \text{and} \quad h = h_c \ \text{(critical thickness)}$$

$$t = \frac{55}{981}\left(\frac{V\mu^5d^9}{\rho g\gamma^4\phi^2h^7}\right)^{1/4} \qquad (13)$$

Then

Prediction Procedure

1. As dispersion band forms from drops of diameter d_0 continuous phase is trapped in the amount expressed by Equation (9).
2. Initial film thickness is estimated from Equation (10).
3. Drops pass into the band and continuous film drains until film thickness becomes h_c (approximately 2.0μ).

Time taken for draining to occur is estimated from Equation (13).

4. Since V is known, the thickness of 1st increment of band is $H_1 = Vt$.
5. On coalescence of a pair of drops. d:ϕ:V:r_0 change and are recalculated for the next increment. Then steps 1 to 5 are repeated until the initial Q_u is reduced to zero.
6. Band thickness is ΣH_i.

A computer program was formulated to carry out the iterative calculations. The calculation procedure was started by assuming that the dispersed phase flow rate in the flocculating zone was 0.74 that is, Ostwaldt's ratio for face centered cubic packing. This is considered to be realistic from observations of the zone, that the drops were very near spherical, and had formed a stable configuration. Thereafter the drops were simply squeezed together forming a dodecahedra confirming that 12 drops surrounded each drop. Initially the critical film thickness was set at 2.0μ and the program processed. Then the value of the critical film thickness was adjusted by 0.5μ increments until the predicted band thickness matched the experimental thickness. This film thickness was then used to predict the band thickness at other flow rates for each system. The comparison between the experimental and predicted band thickness is presented in Figure 9 for the system n-hexane.

There it will be seen that the agreement between the experimental band thickness and the predicted thickness is very good for each flow rate. Furthermore, the critical film thickness computed for each system agreed to within 0.5μ of that estimated by light transmission in the Absorptiometer.

Finally, in order to confirm the validity of the derivation of the model, Christianson's effect was used to photograph the plateau borders and to measure the radius of the arc making up the walls of the plateau border. The measured radius was compared with the predicted value obtained by solution of Equation (11). The plateau borders were photographed with a cine camera because the drops in the bed were in motion so that it was not possible to be absolutely certain that the plateau border being photographed and the camera film were absolutely parallel. The slightest deviation would greatly distort the photographed plateau border radius. A photograph of the plateau border is shown in Figure 10. This is an enlargement of a 2.00-mm square field corresponding to the diameter of the collinated laser beam. The mean value of the radius r_0, measured from 24 consecutive frames of the movie film for the system and taken at a plane 4 cm from the band entrance when the dispersed phase flow rate was 0.67 $\text{cm} \cdot \text{s}^{-1}$ and the total band thickness was 12.0 cm was 3.1 mm. The value predicted from Equation (11) was 3.9 mm, and this is considered to be exceptionally good considering the difficulties in making the measurements. Similar quality agreement was obtained from the other estimations. Therefore this is considered to further confirm the soundness of the model and together with Figure 10 confirms the reliability of the procedure of basing the prediction of the thickness of deep dispersion bands on a critical film thickness and a frequency of coalescence curve. This is considered to be more reliable than attempting to analyze bed thickness from a drop-drop coalescence time for the bed as a whole and for each system.

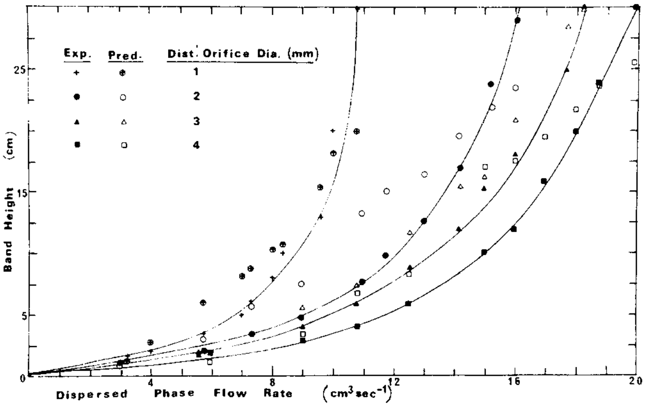


Fig. 9. Comparison between experimental and predicted band height for system *n*-hexane.

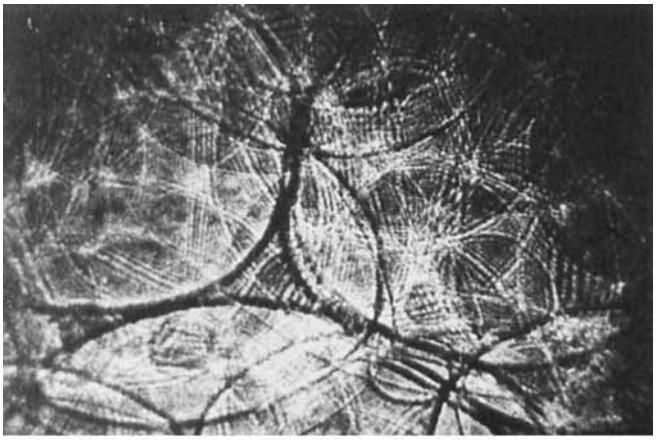


Fig. 10 Photograph of a plateau border inside the dispersion band.

In concluding the discussion, it should be mentioned that no secondary drops of the dispersed phase were seen at any time although these have been observed by other workers in bands and by ourselves in single layers of drops. However, with the experiments in which the phototropic dye was in the continuous phase, it was noticed that very small blue satellite drops appeared above the interface in the coalesced dispersed phase. An allowance was made for this loss in the computer program. Nevertheless it is a finding not previously confirmed and should be considered in the design of phase separating vessels.

NOTATION

A = area of plateau border
d = drop diameter
D = hydraulic mean diameter
F = force causing drainage
g = acceleration
h = film thickness
H = total band thickness
K = constant in Lenard-Lemlick equation
l = length of side of face of dodecahedra
N = number of drops
P = pressure in film
q = incremental flow rate
Q = continuous phase flow rate
r = radius of plateau border
R = radius
t = time
U = drainage velocity
V = superficial velocity
z = coordinate perpendicular to film
γ = interfacial tension
θ = angle of inclination of plateau border
λ = frequency, or probability of coalescence
ϕ = hold-up

Subscripts

c = continuous phase
d = dispersed phase
downward flow
0 = plateau border
u = upward flow

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